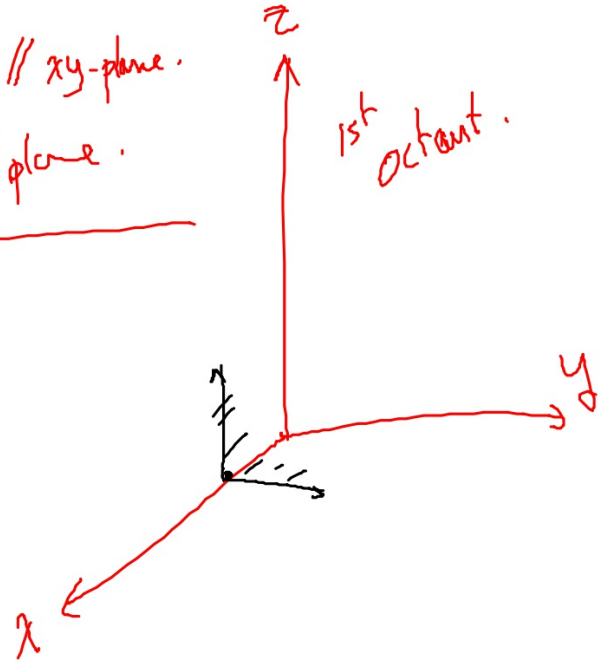
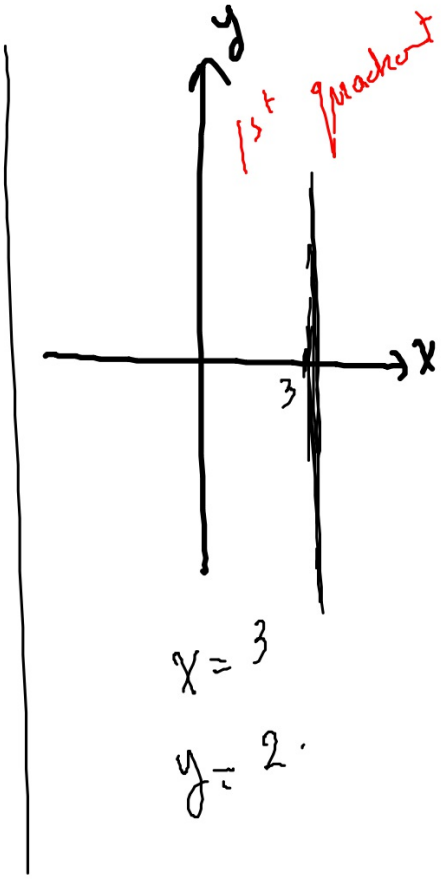
$z = 4$ plane // xy-plane.

Horizontal plane.



1st octant.

$x = 1$



1st quadrant

$x = 3$

$y = 2$.

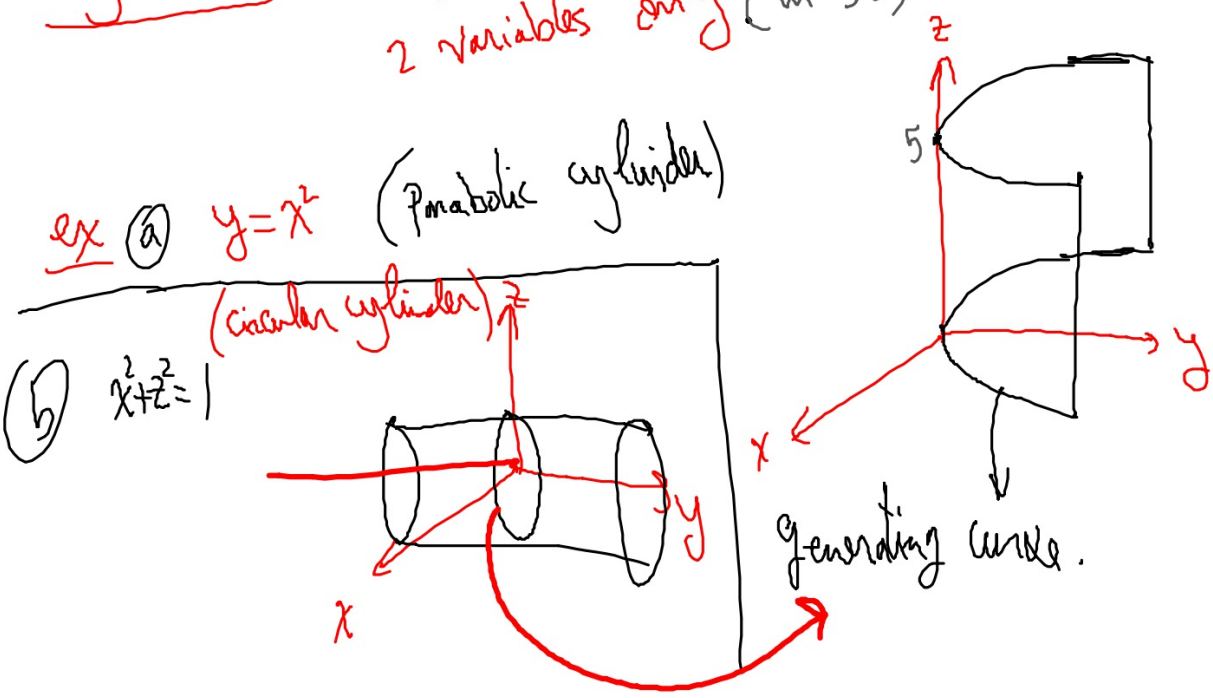
Surfaces needed for MAT 224.

0. special planes ($x=k$, or $y=k$, or $z=k$).

1. <u>Cylinder</u>: the graph of an equation with 2 variables only (in 3D)

<u>ex</u> (a) $y=x^2$ (parabolic cylinder)

(b) $x^2+z^2=1$ (circular cylinder)



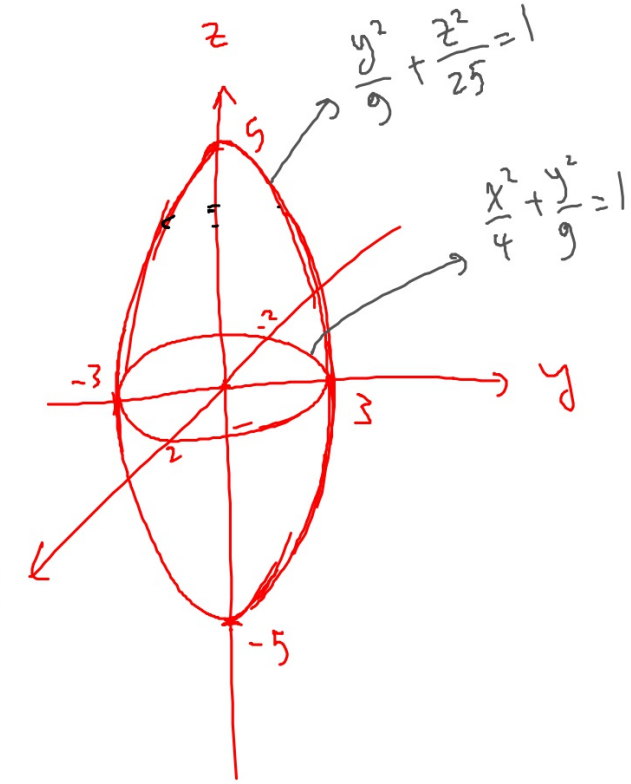
2. Ellipsoids: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$; $a,b,c>0$

3. Elliptical paraboloid: $+\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$
around the z-axis.

4. Hyperbolic Paraboloid: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{z}{c}$ or $\frac{y^2}{b^2}-\frac{x^2}{a^2}=\frac{z}{c}$.
"saddle"

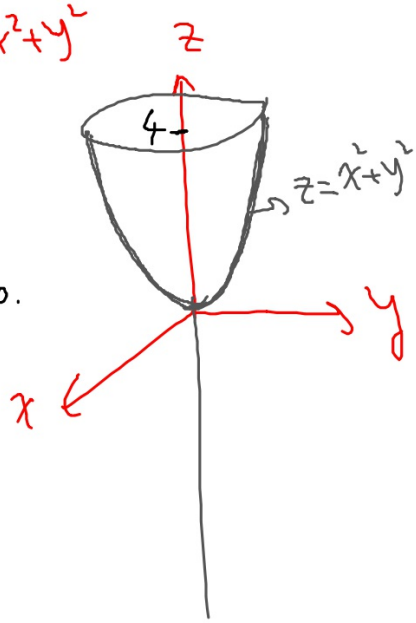
5. Cones: (Elliptical) : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$
around the z-axis

ex: $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 1$

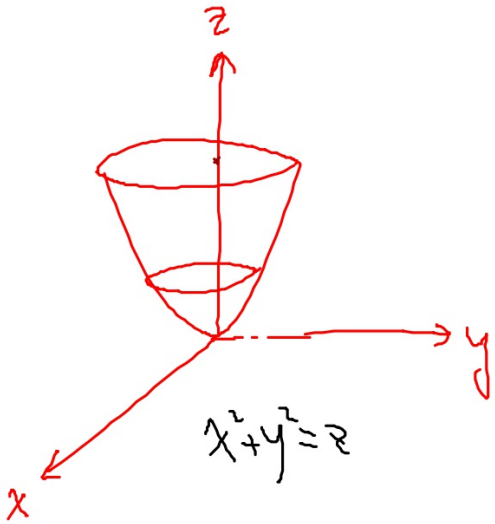


ex $z = x^2 + y^2$

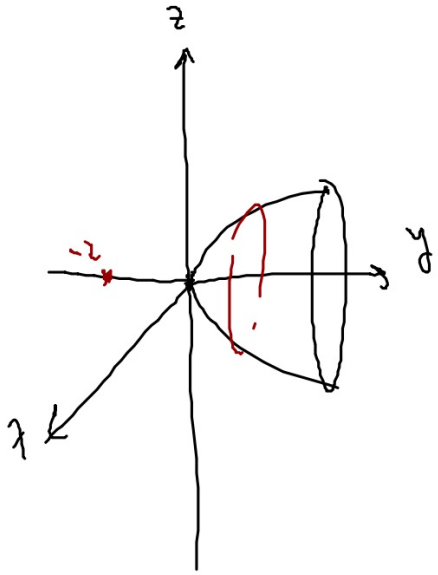
Elliptical Paraboloid around z-axis.



ex (a) $x^2+y^2=z$



(b) $y=\frac{x^2}{4}+\frac{z^2}{9}$



$z = \sqrt{x^2+y^2}$

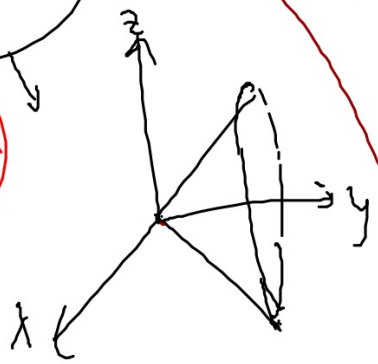
ex

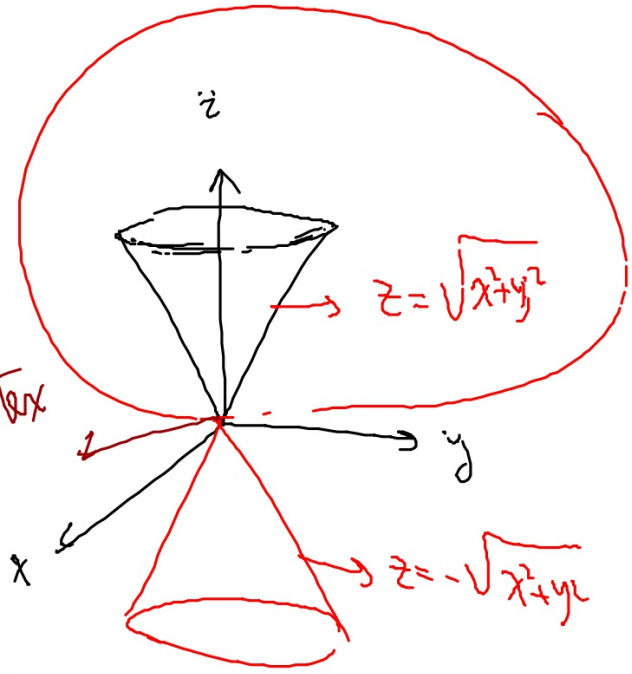
$z^2 = x^2 + y^2$ e $z \geq 0$

$y = \sqrt{x^2+z^2}$

$y^2 = x^2 + z^2$

$y \geq 0$





$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

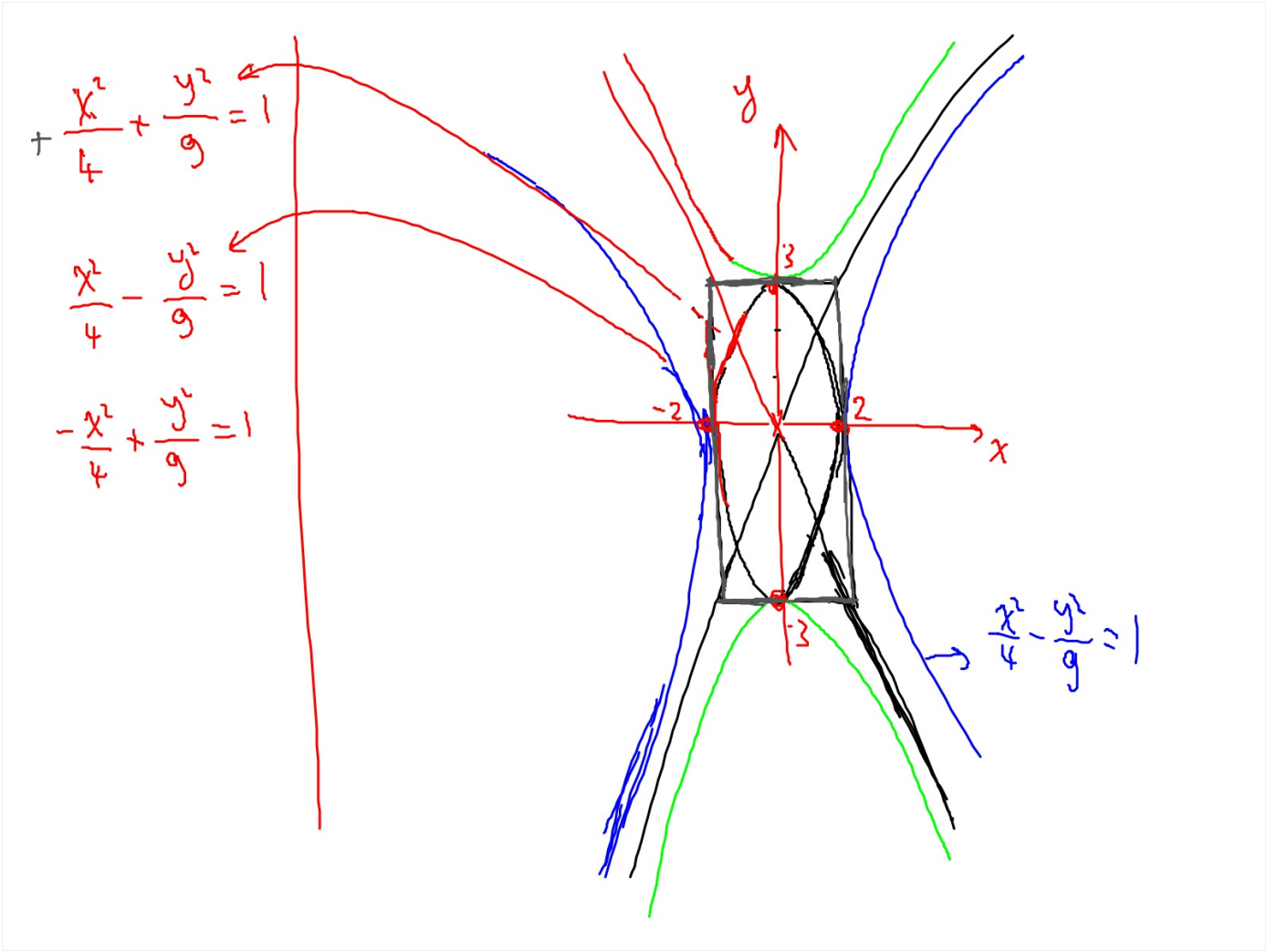
$$z^2 = \frac{c^2}{a^2}x^2 + \frac{c^2}{b^2}y^2$$

If $a=b$ we get

$$z^2 = \frac{c^2}{a^2}(x^2+y^2)$$

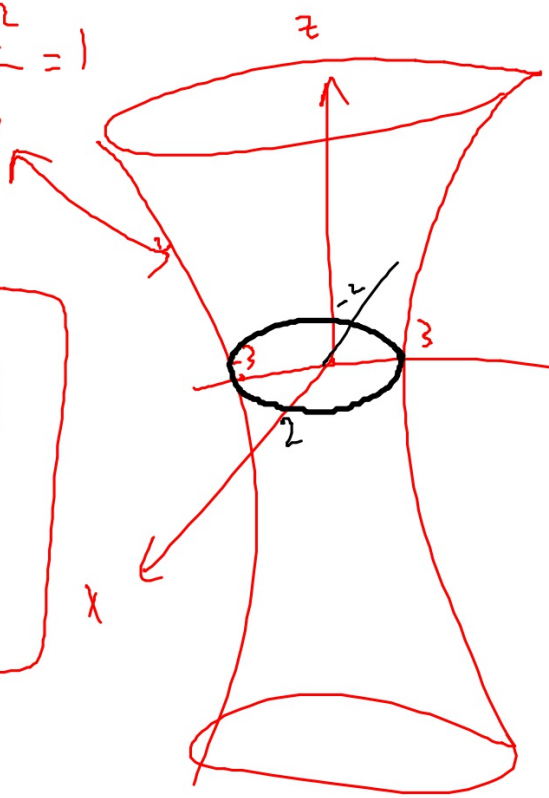
$$z = \pm\frac{c}{a}\sqrt{x^2+y^2}$$

all circular cones around the z axis with vertex at the origin.


$+\frac{x^2}{4}+\frac{y^2}{9}=1$
$\frac{x^2}{4}-\frac{y^2}{9}=1$
$-\frac{x^2}{4}+\frac{y^2}{9}=1$
y
x
3
-3
2
-2
$\frac{x^2}{4}-\frac{y^2}{9}=1$

ex.

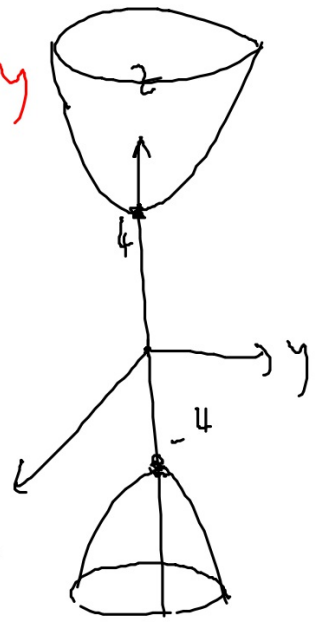
a) $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$



b)

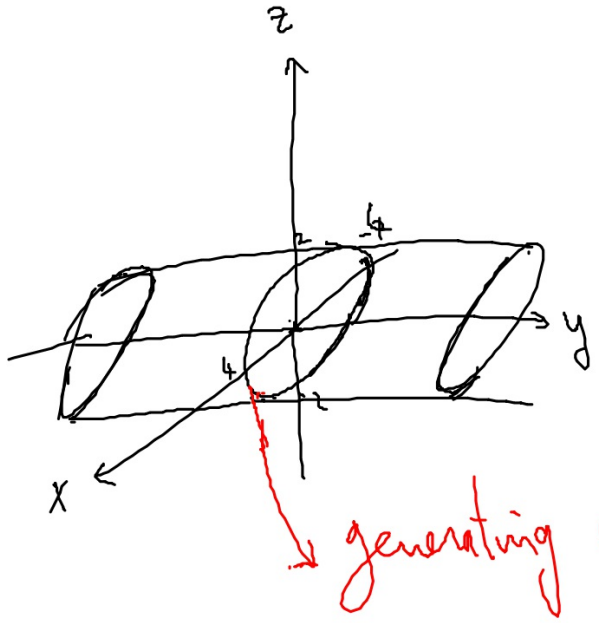
$\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = -1$

$-\frac{x^2}{4} - \frac{y^2}{9} + \frac{z^2}{16} = 1$



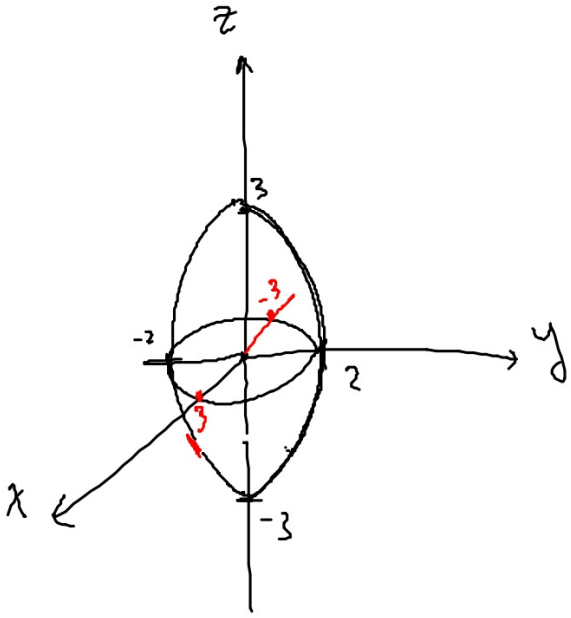
15 $x^2 + 4z^2 = 16 \Rightarrow \frac{x^2}{16} + \frac{z^2}{4} = 1$

Elliptical cylinder around the y-axis



19 $4x^2 + 9y^2 + 4z^2 = 36$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{9} = 1$

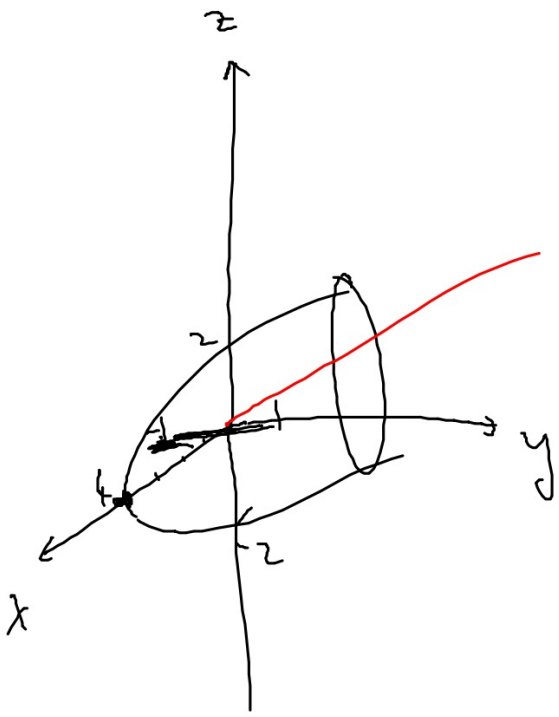


ellipsoid

23 $x = 4 - 4y^2 - z^2$

$(x-4) = -(4y^2 + z^2)$

"Elliptical paraboloid"
around the x-axis



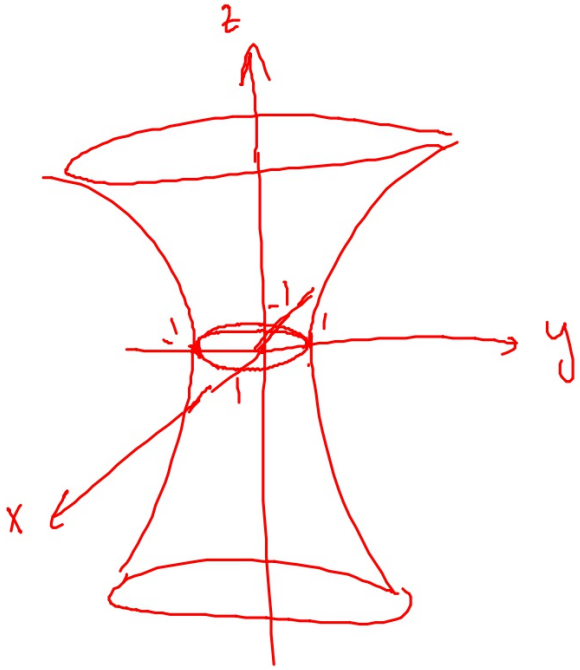
#26 $4x^2 + 9z^2 - 9y^2$

$$\frac{x^2}{9} + \frac{z^2}{4} = \frac{y^2}{4}$$

Elliptical cone around the y-axis.

27 $x^2 + y^2 - z^2 = 1$

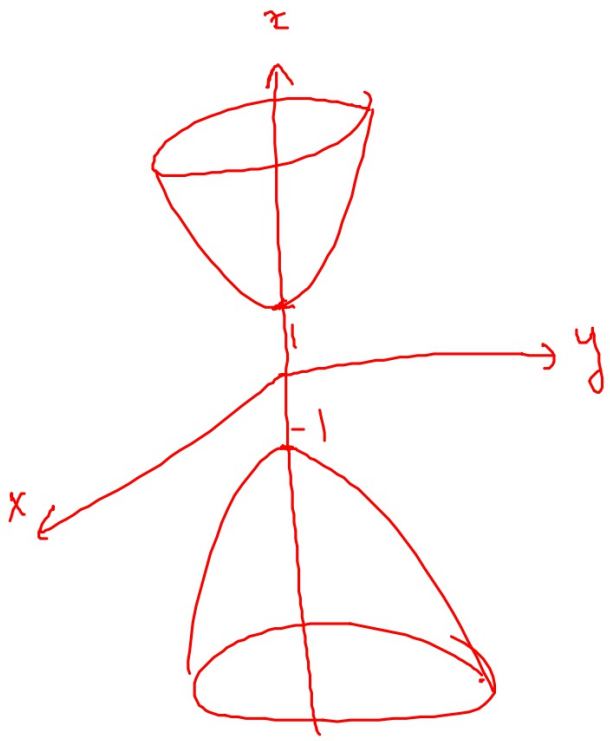
Hyp. one sheet around the z-axis



29 $z^2 - x^2 - y^2 = 1$

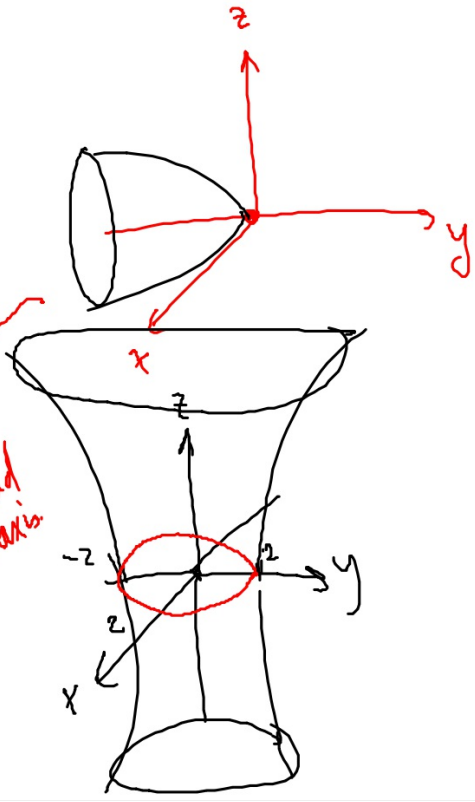
Hyp. 2 sheets

around z-axis.



35 $y = -(x^2+z^2)$

"elliptical paraboloid" around the negative y-axis



37 $x^2+y^2-z^2=4$

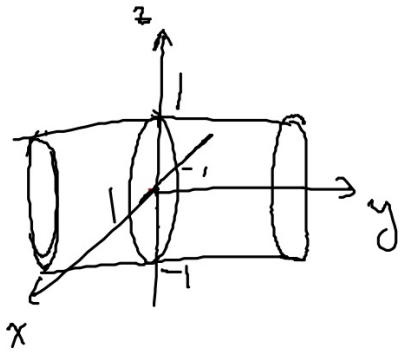
Hyp. one sheet around z-axis

$\frac{x^2}{4}+\frac{y^2}{4}-\frac{z^2}{4}=1$

39 $x^2+z^2=1$

cylinder

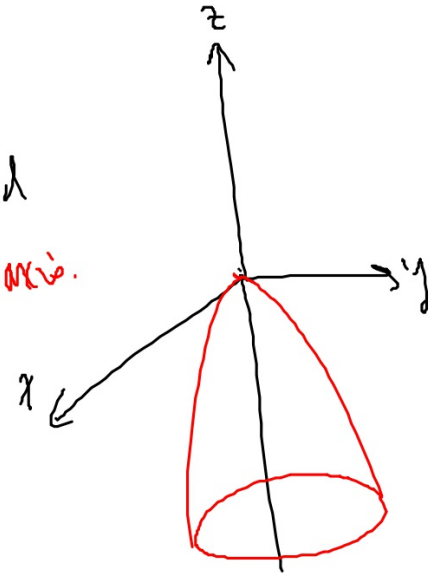
around y-axis



(41) $z=-(x^2+y^2)$

Elliptical paraboloid

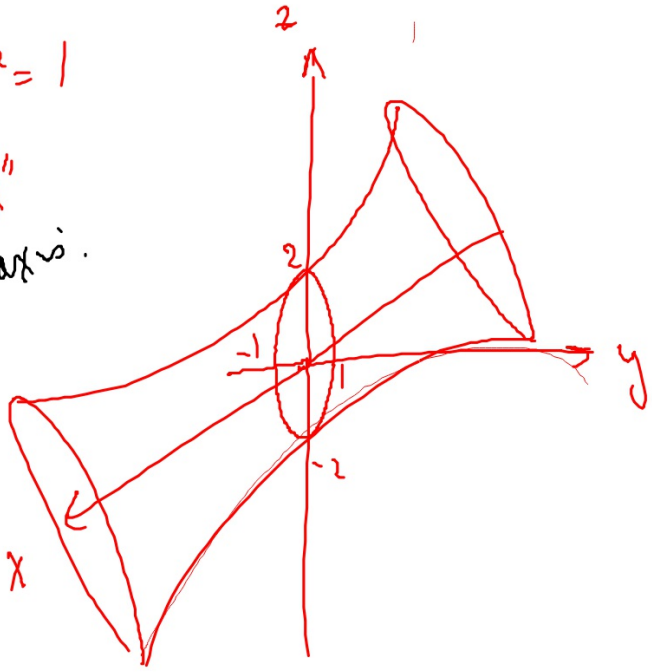
around the Neg z-axis.



43 $4y^2 + z^2 - 4x^2 = 4$

$$y^2 + \frac{z^2}{4} - x^2 = 1$$

= Hyp. one sheet" around the x-axis.



Quiz #1

Identify and graph the following surfaces.

a) $x^2+(y-1)^2=1$ with $z \geq 0$.

b) $z=\sqrt{2x^2+2y^2}$

c) graph both surfaces simultaneously.